Chapter 10
A Whole New Perspective: Special Relativity

Now that I've placed so many appetizers in front of you, here finally comes the main course! It is my hope to have whetted your appetite and stretched your stomach – that is, your mind – adequately to prepare you for what follows. Modern physics, which emerged in the beginning of the twentieth century, held two great surprises for mankind: quantum theory and relativity. Relativity seems to me the less mysterious of the two, but every bit as astonishing. It has captured the popular imagination but does not seem well understood. Einstein presented the theory of relativity in two parts: first came Special Relativity, or relativity in the special cases of uniform motion. This was followed by General Relativity, which covered not only uniform motion but accelerated motion as well. We'll introduce Special Relativity first, with its strange predictions, intimidating mathematics, and shocking philosophical implications. In this chapter I hope to make Special Relativity appear less strange by showing its consistency and symmetry both within itself and in comparison to the other subjects covered so far; in the next chapter I will outline a less threatening mathematical perspective and defuse some of the more dangerous philosophical ideas which Relativity has fueled.

The crux of Special Relativity is the law that all observers will measure light in open space to have the same speed, no matter what their speeds are relative to each other. This means that at relative speeds near the speed of light, two observers' measurements of the space and time between events will differ so that their measurements of the speed of light remain constant. If observer A is in a space station and observer B flies by at 100,000 miles per second in his rocket ship, firing his forward laser guns, the light from the laser guns will have a certain speed. To observer B, the light goes at 186,000 miles per second. To observer A, the light will not have the simple sum of this velocity and the velocity of the rocket ship (186,000 + 100,000 = 286,000 mi/sec). It must still be measured to go at 186,000 miles per second. As a consequence, each observer will measure the other's clock (or any other apparatus of time measurement) to be running slow, and the other's “yardstick” (or any other apparatus of spatial measurement) to be shortened in the direction of their relative velocity. Indeed, each observer will measure her counterpart to be smaller and to be aging less slowly.

On the face of it, this all seems quite ridiculous, that moving clocks should run slower than clocks at rest and that moving objects should be shorter than when they are at rest. But from Einstein's point of view, these conclusions were inescapable, given the facts known at his time. And relativity has been experimentally confirmed over and over again. So how did we come to this point in the history of science, where common sense is no longer sensible? We will follow Einstein's reasoning, and by the time we are done, we should find our former “common sense” replaced by a better sense of reality.

Let's do a quick review of some of the major points we have covered already:

- Apparent size varies with distance and viewing angle (perspective)
- Apparent spatial order is reversed by 180 degree rotation (the apples on the number line)
- Apparent temporal order varies with location in space (the clock array scenario)

- Time is local, not global. Apparent synchronization cannot be made global; it can be created locally only at the expense of true global synchronization. (the clock array scenario)
- Time is radial, not linear. Apparent time rate (frequency) varies with relative radial motion (the Doppler effect)
- Measured distance varies with relative velocity (Galilean relativity)
- Several quantities vary inversely with the square of the distance from a center point: the area of a sphere; the intensity of light; the forces of gravity, electrostatic force, and magnetism; the angular velocities of a series of points measured perpendicular to the direction of motion.
- Magnetism is caused by movement of electric charges and changes in voltage. Electric force may be measured partially as magnetic force depending on the relative motions of the source and the observer.
- Magnetism, unlike electric charge, is apolar. It only appears polar when circular currents occur within a closed surface and measurements are taken outside that surface.
- Circular motion (revolution) and spin (rotation) are equivalent as forms of angular motion.
- Sustained angular motion is represented by a pair of sine waves. Each of the waves represents alternation between two opposites; each pair of opposites relates closely to the other. Examples are: front-back and top-bottom; left-right and clockwise-counterclockwise; north-south and east-west; positive-negative divergence and positive-negative curl; positive-negative one and positive-negative *i*.
- The very concepts of space, time, and causality require a universal maximum “speed limit.” This speed happens to be the speed of light.

As noted above, we know that relative displacement affects *perception* of size. Relativity requires that relative velocity affects the *measurement* of size. We have seen how relative spatial placement will affect whether certain objects are measured to be arranged one behind the other, right-to-left, or left to right. As we shall soon see, Relativity requires that relative velocity will affect whether certain events (A and B) are measured to be simultaneous, A before B, or B before A.

The greatest misconceptions growing out of Relativity concern the nature of time. Einstein's former mathematics professor Hermann Minkowski proposed a geometrical-mathematical model for relativity. Einstein accepted Minkowski's ideas and thereafter described time as a fourth dimension in addition to the three dimensions of space. Whether events were measured to be simultaneous or one after the other was said to depend on one's placement and orientation in a four-dimensional *spacetime.* This model of the universe leads one to imagine that time is somehow at right angles to Cartesian space and that all of history exists in progressively greater distances along a “time axis.” This model is suggested by the light cone diagrams we used in chapter two. This is also what H.G. Wells had suggested in his 1895 novel *The Time Machine.* The 1884 story *Flatland* also suggested the possibility of unseen dimensions.

The Time Traveler in Wells' novel reasons that if time is a dimension, he should be able to move in it as he is able to move through the three dimensions of space. He builds a machine which allows him to do this, visits the far-distant future, and then returns to his own time. Many time-travel stories have been written, because it makes entertaining fiction. But the idea of time as a dimension has deep philosophical consequences. In the tradition of the Greek dialogues, imagine the following conversation between philosophers Bill and Ted:

> **Bill**: Ever since seeing the movie *The Time Machine*, I've wanted a time machine of my own.
> **Ted**: Totally impossible, dude. Time isn't like the street between here and the Circle K. You can't just go back and forth.
> **Bill**: But Einstein said time is a fourth dimension in addition to the three dimensions of space. It's all the same thing, a four-dimensional spacetime.
> **Ted**: Yeah, but what if you went back in time and, like, killed your own parents before you were born? That would create a paradox. I read something like that in *Fantastic Four*. Mister Fantastic tells the Beyonder that he can't kill Doctor Doom because it would create a time paradox and destroy the whole space-time continuum or something.
> **Bill**: But what about *Deja Vu*? Maybe if you go back in time, you still can't change anything, just like what's-his-name, Denzel Washington.

Einstein's theory of relativity has its roots in the work of James Clerk Maxwell. Maxwell's equations explained light as an electromagnetic wave. A long-standing question then became even more important: If light is a wave, what is the medium that sustains the wave? Without water, there are no water waves. Remove the air from a test chamber containing a tight balloon, and there is nothing to carry the sound waves when the balloon pops. All waves have to be a disturbance of *something*. The idea of a “luminiferous” (light-bearing) “ether” was invented to give light some sort of medium to travel through in the otherwise empty space between the stars. The ether would be the single frame of reference in which light would travel at the speed defined by the Maxwell equations. Anyone in motion relative to the ether, it was reasoned, would find light to have a slightly different speed. Several attempts were made to detect this ether, all of which failed. The Michelson-Morley experiments of 1887 were ingenious, and today we consider them definitive and conclusive: they showed that there is no ether and no such universal reference frame. The question of the medium of light thus becomes bigger. The Maxwell equations require that light travels at a certain speed; what is that speed relative to if not to some medium?

Einstein's answer was that the speed of light is relative to whomever is making the measurements; it must be the same for all observers. Michelson and Morley had proven that there was no absolute frame of reference defining the speed of light. To suppose, then, that one observer would measure a different speed of light than another was to say that one observer's results conformed to Maxwell's theory of light more closely than another's; that there *was* a preferred frame of reference and that Michelson and Morley had got it wrong. Einstein's radical theory was the inescapable consequence of unassailable reasoning and experimental fact. And once you accept Einstein's premise, you must accept all that follows from it, however astonishing. As the correctness of Einstein's premise has been explained quite effectively by many others, I will not belabor the point further here. To the readers who are not convinced, I have recommended some resources in the bibliography.

Before going more deeply into the strange things predicted by relativity, it is important to make and repeat some distinctions. Appearance is different from measurement. In chapter two, we made this clear with the chimes and pistols sounding or appearing to be out of sync when they actually were synchronized. Also, we mustn't confuse places with events. A place is a location in space. An event is a specific place at a specific time.

While trying to reconcile the ether theory to the experimental failures in detecting the ether, and before Einstein made the ether theory unnecessary, H. A. Lorentz theorized that an observer's motion through the ether actually caused him and his instruments to shorten in the direction of motion. That, he supposed, was why the observer always measured light to have the same speed in all directions. Lorentz's theory is no longer useful, but the equations he developed to support it are still in use. They are somewhat unsightly, but Einstein found them ready-made for relativity and they do indeed represent the degree to which one observer finds the measurements of another observer in relative motion to be lengthened in the direction of motion. According to relativity, the observers themselves do not merely *appear* but are *measured* to be proportionally smaller. This is how they can both agree on the speed of light.

The "Lorentz factor" is complex enough to be abbreviated a number of ways. Sometimes it is symbolized by the Greek letter gamma (γ). Sometimes it is shortened by using the letter beta (β) to stand for the ratio of relative velocity to the speed of light ($\frac{v}{c}$). In this form, the Lorentz factor is : $\frac{1}{\sqrt{1-\beta^2}}$. Unabbreviated, it takes the form: $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ **(Equation 10-1)**.

Notice that when v is zero, or at least much less than c, the Lorentz factor is one or near one. Multiplying anything by one has no effect; that means that at low speeds, the Lorentz factor does not come into play. But as v reaches significant fractions of c, the Lorentz factor becomes large enough to notice. As v approaches c, the factor gets very large indeed, and where $v=c$ it results in division by zero, which is undefined. This reflects the fact that nothing other than light (or other forms of electromagnetic radiation) may go at the speed of light.

Table 10-1. The Lorentz factor at some sample velocities.

v (km/s)	v/c	Lorentz factor
0	0	1
1000	~0	1
50,000	.16	1.01
150,000	.5	1.15
225,000	.75	1.51
270,000	.9	2.29
297,000	.99	7.09
299,700	.999	22.37

Do not let yourself be distressed over the apparent complexity of the Lorentz factor or the Lorentz formulas which follow. Part of my interest in physics is the beauty and simplicity of its mathematical patterns. If I thought that the expression $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ occurred in nature and could not be rewritten in simpler terms, I might be tempted to quit the study of physics. I will propose a more derived (that is to say, less direct) but simpler form later on.

With the Galilean transformations which we learned in chapter five, we transform the measurements of two coordinate systems in uniform relative motion in this way:

x'=x-vt
t'=t

If someone else has velocity v along the x axis of my frame of reference, I use the above equations to reconcile my measurements with hers. We did this exercise in chapter five with the dropping ball. At low speeds, that is, speeds much less than the speed of light, the Galilean transformations work just fine. This is because the $\frac{v}{c}$ term, β, in the Lorentz factor becomes zero, making the factor as a whole equal to one. But at higher speeds, where $\frac{v}{c}$ becomes larger, the Galilean transformations no longer work if both observers are to measure light to have the same speed in all directions. For high speeds, the Lorentz factor (γ) must be included:

Equation 10-2. $x'=\gamma(x-vt)$

Equation 10-3. $t'=\gamma(t-\frac{vx}{c^2})$

Equation 10-2 shows how the distance x' between two events as measured by one observer will differ from the distance x measured by an observer in relative motion along the x axis at velocity v. If the observer who measures a distance of x between the events also measures them as being simultaneous (t=0, therefore also vt=0), then the observer in relative motion will measure a distance of x' which is simply x times the Lorentz factor.

Equation 10-3 shows how the time t' between two events as measured by one observer will differ from the time t measured by an observer in relative motion along the x axis at velocity v. If the observer who measures a distance of x between the events also measures them as being simultaneous (t=0, therefore also vt=0), then the observer in relative motion will measure a time difference of t' between the events; in other words, they will not appear to be simultaneous.

Here is an example. You and your friend drive cars which are both fifteen feet long. When you are stopped side by side or traveling at the same speed, each of you measures the other's car to be fifteen feet long. Using the above formula, the Lorentz factor is one when your relative velocity (v) is zero.

Let's suppose that the speed of light were only 60 miles per hour (about 44 feet per second). You are stopped at a traffic light, but your friend comes up alongside you just as the light turns green and passes you at 30 miles per hour (22 feet per second). The Lorentz factor for this relative speed, half the speed of light, is 1.15, as shown in Table 10-1.

So if each of you now were to measure the distance between two events which are simultaneous in your frame of reference, the other would find the distance between those two events to be only 15 percent larger; and the two events would not be simultaneous in the other observer's frame of reference.

Because each of you measures the front and rear ends of the other's car at offset times rather than times which are simultaneous to the driver of the car, the other's car is measured to be shrunken along the direction of your relative motion. At relative speeds of one-half the speed of light, you measure your friend's car to be $1/\gamma$ or 86.6 percent as long as your own.

While you are stopped at the traffic light, suppose you have your hazard lights blinking. To you, the lights in the front and rear of your car blink on and off at the same time. But to your friend, they do not blink simultaneously. Your front lights lead your rear lights slightly, even as he comes alongside you. If your friend has his hazard lights on, you will measure the same thing, but in reverse. His rear lights lead his front lights. There is a fundamental reality upon which both of you will agree. You both agree on the speed of light. You also will agree not on the length of your cars, namely the spatial distances between the two places of “front” and “rear,” but on the combined space-time distance between the *events* associated with these places. You will agree on the combined space-time distance between the particular events of the individual lights blinking on or off.

Let's establish a three-dimensional coordinate system to better analyze the problem. The street you and your friend are traveling on will be our x axis. The intersecting street is the y axis, and a line perpendicular to the plane of the intersection is the z axis.

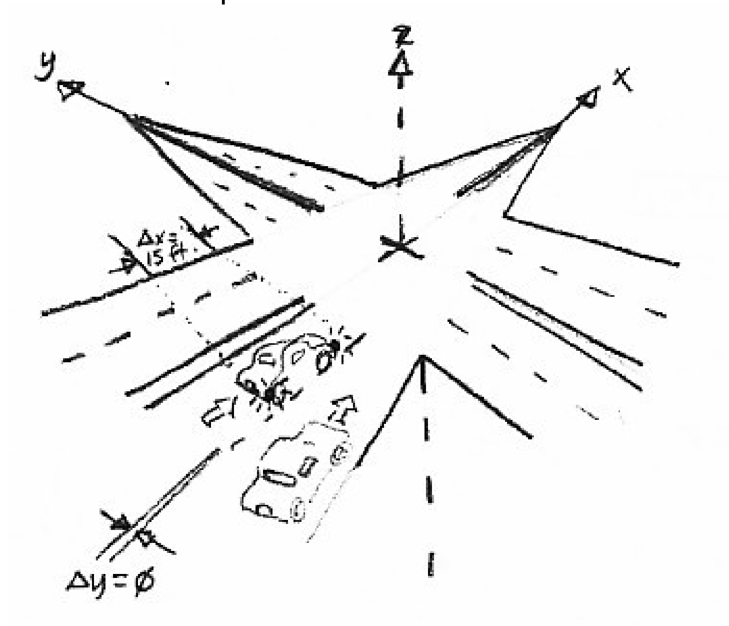

Figure 10-1. A street intersection with three Cartesian axes. Lengths Δx and Δy between two blinking hazard lights.

Let's focus on just one blink of your hazard lights, and just on the right-hand side of your car where your friend passes by. The blink of the right front light is one event, and the blink of the right rear light is another.

For you, the distance between the two events is 15 feet along the x axis. You measure no difference along the y or z axes, and no difference in time.

For your friend, the difference in time is about two tenths of a second, which means he is measuring the events at two different times. It works out that the distance between the two events is 17.3 feet in his moving frame of reference. The two (simultaneous) ends of your car he measures at 13 feet apart, but since the events of the blinking lights are not simultaneous to him, he has moved slightly in the interim and this extends the measurement in his frame of reference. Though he finds the length of your car – the distance between two *places* – shortened by the Lorentz factor (the length of your car is multiplied by $1/\gamma$), he finds the distance between the two *events* to be *lengthened* by the Lorentz factor (length is multiplied by γ). If each of you had large clocks on the ends of your cars and each of you synchronized your own clocks, you would see a time difference between the two clocks on the other car. You each measure the other car to be shorter because from the other person's perspective you are measuring the leading end of their car first and then the trailing end later, even though in your frame of reference you are measuring both ends simultaneously. How do we mathematically reconcile your measurements to those of your friend? With the rules of geometry.