Chapter 5
Perspective in Motion: Galilean Relativity and the Doppler Effect

Much can be – and has been – written about the astronomer, mathematician, and physicist Galileo Galilei (1564-1642). Galileo's was one of the most influential minds in human history. He is generally credited with the observation that the acceleration of gravity is independent of an object's mass. He was the first to discover moons orbiting a planet other than the earth. He helped promote the idea that the planets revolve around the earth and not the sun. He made improvements on the telescope and the compass.

The particular legacy of Galileo which I will present now concerns how we reconcile one observer's set of measurements to another's, particularly when one of those observers is in motion. Imagine yourself standing and holding a ball over your head, then dropping the ball to the ground. It does not fall to your left or right, but six feet straight down. It takes one and a half seconds to hit the ground after you release it. Now imagine that you are on the back of a flatbed truck headed south at seven feet per second, or about 5 miles per hour. You repeat the same action. The ball falls six feet straight down and hits the truck bed 1.5 seconds after you release it. But Jane is standing at the side of the road and measures the same events. She also measures that the ball took 1.5 seconds to fall. And she agrees that the truck bed was six feet below your hand, and therefore the ball was six feet lower when it reached the end of its fall. But from her point of view, the ball also strikes the truck bed about ten and a half feet further south from where it left your hand. You say the ball fell neither left nor right; she says it fell 10.5 feet to the right.

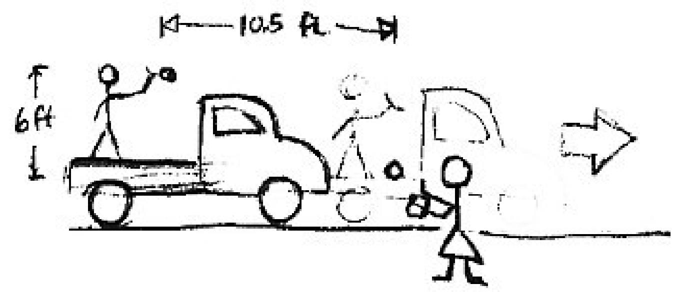

Figure 5-1.

Galileo gives us a simple formula for reconciling these two measurements. Let's call our direction of travel (southbound in the truck) the x axis in our Cartesian coordinate system. In this case, we share the x axis (the road) with Jane, our roadside observer, but not the origin. We are at the center of our system and she is at the center of hers. Since we are moving relative to one another, our measurements along the x axis disagree more and more over time. Galileo's formula for making our measurements match is simple. We are measuring the difference between two events: letting go of the ball and the ball striking the truck bed. To figure out Jane's x measurement between the two events, I simply take mine and add to it. What I add is the time I measured between the two events, multiplied by the speed of my truck. The formula is:

Equation 5-1. $x' = x + (vt)$

x′ ("x *prime*") is Jane's distance measurement. *x* is ours. *v* is the velocity, or speed, of the truck, and *t* is the time measurement.

So Jane's distance measurement is our measurement (zero) plus the extra 7 feet per second times 1.5 seconds. Putting it all together, we get 0 + (7 x 1.5) = 10.5 feet. Jane can do the same exercise from her point of view. To find out our distance measurement, she takes hers (10.5 feet) and adds the velocity of the truck times 1.5 seconds. To her, the velocity of the truck is opposite in value, a "minus seven" rather than seven. This is because from her viewpoint the truck is moving south, but from our viewpoint she is moving north. Doing the math, Jane gets 10.5 + (-7 x 1.5) = 0.

This formula works for any situation. We could also imagine two people playing ping pong aboard an eastbound bullet train. From their point of view, The ball goes east, then west, then east again, and so on. From the point of view of someone standing beside the track, the ball may always be going east, but it is going faster, then slower, then faster, and so on. Each point of view is valid and each can be reconciled to the other using Galileo's formula. Such a point of view is called a *frame of reference* in physics. A frame of reference is more or less the same as a coordinate system, as we discussed in chapter four, but with an important distinction: if two frames of reference share the same coordinate system at one particular moment but one is *moving* relative to the other, their measurements over time will differ.

We used Galileo's formula above to do mathematical *transformations* from one frame of reference to another. An important assumption underlying Galilean transformations is that we and Jane will measure the same interval of time between any two events. Mathematically, Galileo's rule was:

$t'=t$

This was a reasonable assumption, and without it, the transformations would have been more complicated. It is even a valid assumption at low speeds. But this assumption begins to fail us at high speeds in the same way that flat maps fail us over large areas of curved space. We call the Galilean transformations the formulas for "Galilean relativity." These formulas tell us that the distance measured between two events depends on the relative speeds of the observers. Einstein's relativity takes this idea further, undoing the assumption of universal time in the process.

Though few may see it as such, the first attack on the idea of a universal time may have been a theory published in 1842 by Christian Doppler and experimentally confirmed three years later. What we now call the Doppler effect is the change in frequency which results from movement toward or away from a wave source. We hear this effect in sound waves all the time. As our truck approaches Jane, suppose she hears the music from our radio. As we pass by, if we are moving fast enough, she will notice that the music seems to change keys. It sounds lower after we pass. The roar of our tires on the pavement may change in pitch as well. The Doppler effect is heard in the whistles and horns of passing trains and cars, and it is even seen in the light of distant stars. If we think of the rising and setting of the sun as a wave, then Mr. Fogg experienced the Doppler effect in the frequency of the sun's movements as a result of his eastward travel.

A wave is a series of events. Water rises and falls. Air vibrates back and forth to create sound. The *frequency* of a series of events is the inverse (or shortness) of the time between events. This is written as:

$$f = \frac{1}{t}$$

The time frequency, *f*, of a wave is one divided by *t*, the duration between wave crests. If you stand near a pond in a high wind and notice that a ripple reaches the shore every two seconds, then you could say that the waves in the pond have a frequency of ½. The water level at the edge of the pond is rising, falling, and and rising again; this is a repeating cycle which is halfway completed every second. The voltage in North American household power outlets changes direction from positive to negative and back again sixty times every second. This varying voltage can also be visualized as a wave of rising and falling voltage, a cycle that lasts 1/60 of a second. The cycle is completed sixty times per second so it has a frequency of 60. We say “60 Hertz” in honor of the physicist Heinrich Hertz, the first to create and detect radio waves. FM radio frequencies are counted in megahertz, or millions of cycles per second.

We have described how the frequency of a wave, which is the pitch of a sound or the color of a light, is the inverse (or shortness) of its *duration* in seconds. But the frequency of waves which travel at a known speed can also be calculated based on their spatial *length:*

$$v = f\lambda$$

The velocity of a wave, *v*, is equal to its frequency, *f*, times its wavelength (commonly represented by the Greek letter lamba). The speed of sound in air is a fixed value. If the wind is not blowing in a particular direction, then sound waves will travel over the ground at about 1100 feet per second, regardless of how the source of the sound may be moving.

We have already seen that we disagree with Jane on the distance between any two events which happen at different points on the road. Since we must therefore disagree on the length of sound waves traveling along the road, we also must disagree on their frequency. The truck radio appears to slow down slightly in time as it passes, as if an old-fashioned turntable in the radio station were slowing its spin. We disagree with Jane not only on the measurement of one dimension of space, but on one measurement of time as well. We don't hear the music slowing down as Jane does. Galileo showed us how to make our distance calculations correspond to Jane's using mathematical transformations from one perspective to another. He adjusted our moving yardsticks. Doppler recalibrated our clocks.

Here is where your brain may really start to hurt, so if you need some time to catch up, take it now.

Waves are very easily visualized by imagining a stone dropping in a still pond and creating a series of expanding, concentric circles. In this regard, sound waves are much like water waves. Sound waves propagate spherically rather than circularly, but if we ignore what happens in the air above us and just focus on what happens at ground level, the expanding circles are a pretty fair representation. The water waves are caused by the up-and-down movement of the water where the stone has dropped. When the up-and-down movement stops at the center, no more waves are generated. Sound waves are caused by vibrations in the air, like a reed or string moving back and forth. When the reed or the string stops, no more sound waves come from it, although the ones that have already started keep going in their ever-expanding circles until the effect is spread too thin for us to hear it. But when we are getting closer to or further from the source of the sound, these waves seem to get closer or further apart in that particular direction as well.

Figure 5-2

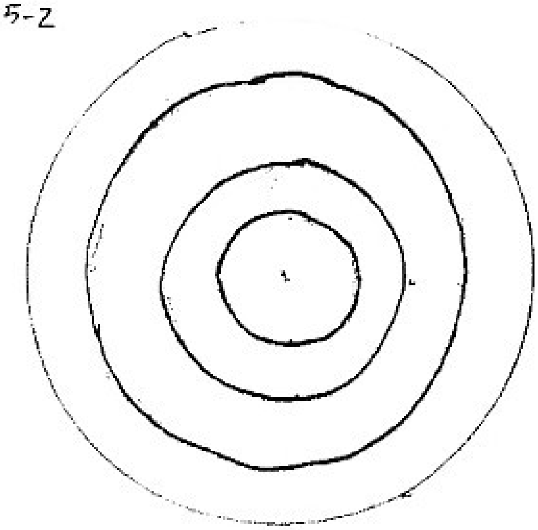

Figure 5-2. Concentric waves.

In Figure 5-2, where there is no relative motion, the circular wave crests are *concentric*, meaning they share the same center. In Figure 5-3, we see what happens when there is relative movement. The circles are no longer concentric because they don't appear to originate from the same place each time. We see that the wavelength on one side is shorter than the wavelength on the other side. This is the pitch of the music on either side of the moving truck. As our relative speed with the wave source approaches the speed of the waves through their medium (for example, the speed of sound waves in the medium of air), the circles meet on one side. When our relative speed is faster than the waves, something new happens. Not only are the circles no longer concentric, but each circle actually begins outside the preceding one. We saw this in our sonic boom diagram in chapter two. We also saw it with the race officials. The sequential firing of each stationary pistol creates a series of events which could also come from a single pistol which fires as it moves down the starting line. If it traveled at the speed of sound, its waves would appear as seen in Figure 2-6. If it moved with near-infinite speed, it would seem to create all the shots at the same time. Each circle would appear outside the preceding one, much like the simultaneous firing diagrammed in Figure 2-3 .

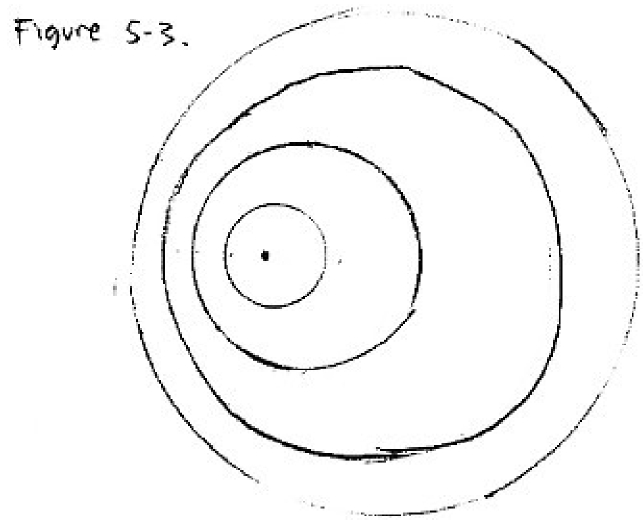

Figure 5-3.

In Figure 5-3, we could imagine that we are looking at a cone which is tilted slightly toward the left. In Figure 5-2 we may see that we are looking straight down the center of the same cone. Do you see it? Do you see the cone pointing toward you, or is it pointed away from you? Maybe it is two cones pointing in both directions, like the light cones we discussed in chapter two. This is a useful mental connection to make. Let's imagine that the cones are pointing away from us and that we are seeing them from the inside. In Figure 5-4, where the wave source is moving faster than the waves, it appears as though we have passed through the side of the cone and are now seeing the tip from the outside. For sound waves, this is possible. But because nothing can move faster than the speed of light, we cannot move outside of the cone made by light waves. As we pointed out in chapter two, we cannot reach what is outside of our future light cone.

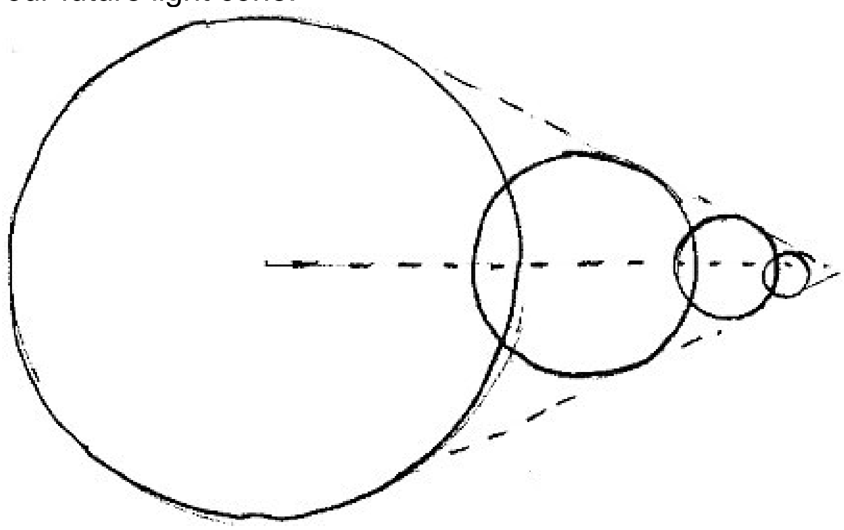

Figure 5-4.

There is an important distinction to be made between the differences we and Jane will measure in the first case of the dropping ball and the second case of the change in pitch of the radio. In the first case, we have differences in measurement due to a difference in *linear* velocity. That is, Jane's motion in our frame of reference is in a straight line, and we are likewise in motion through her frame of reference. Whether Jane is standing right at the roadside or several yards away will not change the distance that she will measure that our dropped ball falls to the right as we pass by her at a given speed. Not so with the change in pitch of the radio. If Jane is standing by the roadside, she will hear a much more noticeable change in in pitch than if she were several yards away. The Doppler effect depends not so much on linear velocity as *radial* velocity. It does not matter how fast we are moving through Jane's frame of reference; it matters how fast we are getting closer or further away from her. To repeat a distinction I will be making throughout this book, space is linear; time is radial.

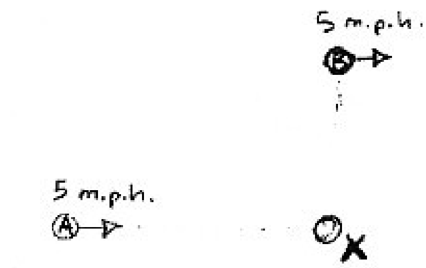

Figure 5-5. Though both A and B have a linear velocity with respect to X of 5 m.p.h., A's radial velocity with respect to X is 5 m.p.h. and B's radial velocity with respect to X is zero at its moment of closest approach.