## Chapter 6
## Perspective in Mass, Force, and Energy

In this chapter we will be looking at some of the fundamental forces of nature: gravity, magnetism, and static electricity. We will compare and contrast them and in doing so we will be discussing the work of Isaac Newton (1643-1727) and Michael Faraday (1791-1867), two scientists who had key roles in bringing about respectively the beginning and end of what we call the classical age of physics.

Isaac Newton is considered by many to be the most influential person in the history of science. His accomplishments and legacy are too far-ranging to fully discuss here, but at least a few key points must be noted. Newton's ideas, by mathematically defining the relationships of motion, force, gravitation, and mass, absolutely dominated the world of science for over two hundred years. He also was one of two mathematicians who independently developed the branch of mathematics we know as calculus. His mathematical techniques and physical laws were so far-reaching that they raised philosophical questions about the role of free will in the universe. It was speculated that if one only knew the position, mass, and velocity of everything in the universe at a given moment, one could predict everything that would ever happen from that moment onward. By relating the downward curve of projectiles (or apples) to the elliptical motions of the planets, Newton showed that the same laws of gravitation operated in the heavens as on the earth. His famous three laws of motion related mass, force, and motion to one another. I will state these three laws in simple terms:

**Newton's First Law:** *A mass at rest or in uniform motion will remain at rest or in uniform motion unless acted upon by some force.* “Uniform” motion means motion in a straight line and at a constant speed. We may to think that everything that moves will come to rest eventually even if left alone. But the reason your car stops when you run out of gas is either the force of gravity (if you are going uphill) or the force of friction between your tires and the road. Put another way, Newton's first law states that no acceleration happens without a force. At rest, you have a velocity of zero and your rate of acceleration is zero. In uniform motion, your velocity has some non-zero value but your acceleration is still zero; your direction and speed are constant.

**Newton's Second Law**: *Force equals mass times acceleration.* Mathematically, this is written as $F=ma$. In some ways, this is a restatement of the First Law: no force, no acceleration. But it goes beyond the first law in defining force mathematically as the product of mass and acceleration.

**Newton's Third Law**: *For every action there is an equal and opposite reaction.* If you push your car forward to get it to the gas station, you are pushed back. If you don't balance yourself by also pushing backward against the road, you will tip right over. You, having relatively less mass, will be pushed backwards by the car quite a bit. Your car, having much more mass, will be pushed forward by you much less. The force is equal in both directions and has the same direction as the acceleration in each case. The car accelerates forward; you accelerate backward. Acceleration is a change in speed or direction and can be positive or negative. If I step on the gas pedal of my car, the engine applies a force which accelerates my car in the same direction as my velocity. I speed up. If I step on the brakes, they apply a force which accelerates the car in a direction opposite of my velocity. The result is a drop in speed. If I turn the steering wheel one way or another, my wheels interact with the road to produce a force which accelerates the car in a left or right direction. Notice that the *car* is accelerated by these actions, not me or any of my passengers. The force which accelerates anyone inside the car comes from the back of the seat as I press the gas pedal, or the steering wheel, or the panic handle above the passenger-side window as I turn a sharp corner. It may come (heaven forbid) from the seat belts or airbags as the car comes to a sudden or violent stop.

Before we go further, we should define our measurements. As you may know, there is more than one system of measurement and this can confuse us somewhat. Let's start with the metric system. Distance is measured in meters (or kilometers, and so on), and time in seconds. Distance has a direction in space. If you talk about distance, you mean a distance from something to something else, and that distance has a direction. Velocity is measured in meters per second. This also has a direction, as opposed to speed, which generally does not. Where velocity is the rate of change in distance, acceleration is the rate of change of velocity. As we have pointed out already, acceleration has a direction as well as a numerical value. Acceleration is measured in meters per second per second, or by the somewhat opaque expression, “meters per second squared.” If you find this confusing, work your way from “velocity per second” to “ meters per second per second.” Dividing something by the same quantity (seconds) twice is mathematically the same as dividing by that quantity squared. This illustrates the point that when we multiply or divide various quantities, we are operating on their units of measurement as well as their numerical values. When we multiply acceleration (meters divided by seconds squared) by time (seconds), the units of seconds “cancel out” once algebraically to give us a velocity, which is measured in meters per second.

$$\frac{5m}{s^2} \times 3s = \frac{5m \times 3s}{s^2} = \frac{5m \times 3\cancel{s}}{s^{\cancel{2}}} = \frac{15m}{s}$$

**Equation 6-1.** If a body accelerates at a rate of 5 meters per second per second for 3 seconds, it gains a velocity of 15 meters per second.

Force, then, could be measured in units of mass times acceleration, or mass times meters per second squared. This would be a mouthful, so instead we measure force in Newtons. Force also has a direction, which is the direction of the acceleration it causes. Mass is measured in grams and (the importance of this next distinction will be apparent later on) it does not have a direction, only a value. Mass is not the same as *weight*. A typical human mass of 100 kilograms (100,000 grams) has no *weight* in space when no force is acting on it. Weight, measured in *pounds* in some English-speaking areas of the world, is an expression of force. Typically it is the force of the ground pressing back on a mass which is being accelerated toward the ground by gravity. But even though a mass in space has no weight, force is required to accelerate it.

In addition to the three laws of motion, Newton gave us a formula for calculation the force of gravity between any two masses. It states that the force of gravity depends on the mass of the two bodies and the distance between them:

**Equation 6-2.** $F = G\frac{m_1 m_2}{r^2}$

The larger either of the masses is, the greater will be the force between them. The term $r^2$ in the formula tells us that the force of gravity varies inversely with not just the distance between the masses, but with the *square* of the distance, In other words, if the distance is twice as great, the force is one-fourth as strong. Cut the distance to one-third, and the force will be nine times greater. The term G is a numerical constant which makes the calculation come out in terms of Newtons. If force were measured on some other scale, G would have a different value or would drop out. It is used the same way pi is used to relate the circumference of a circle to its radius, or the arc length of a circle segment to its angle in degrees (Figure 6-1).

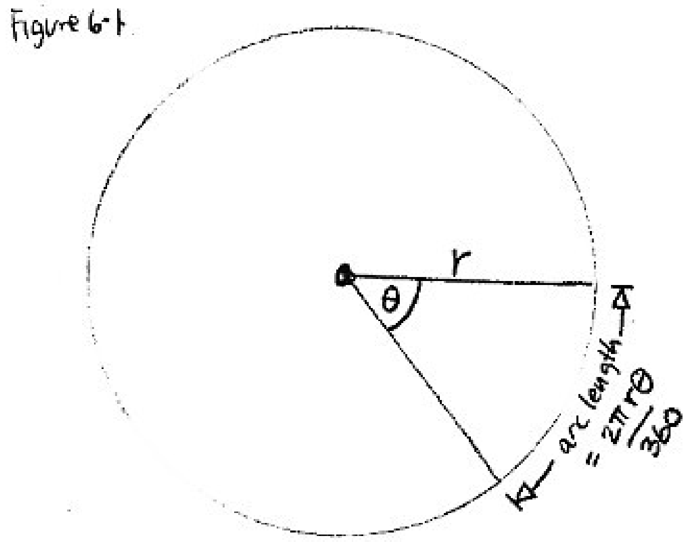


**Figure 6-1.**

When degrees are measured in radians instead of degrees, pi is no longer needed to calculate arc length. The point is that we should focus on the relationship more in terms of mass and distance squared than the constant G.

By combining the law of gravitation with the second law of motion, we discover an interesting result.

If $F = G\frac{m_1 m_2}{r^2}$ and $F = ma$ , then $ma = G\frac{m_1 m_2}{r^2}$

Dividing by mass on both sides of this new equation, we find that $a = G\frac{m}{r^2}$ .

In English, this says that a body of a given mass will accelerate an object at a given distance with a particular rate of acceleration, independent of the mass of the object it is accelerating. Gravity as a *force* (or *weight*) depends on the mass of the object it is acting on. Gravity as a cause of *acceleration* is *independent* of the object it is acting on. For instance, all objects near the surface of the earth will tend to accelerate downward at a rate of 9.8 meters per second per second. Drop a stone down a deep well and after one second it will be dropping at about 10 meters per second. After two seconds, it will be dropping at nearly 20 meters per second, and so on. The mass of the stone does not matter. It seems somewhat mystifying that gravity should be more constant as an acceleration rather than as a force. This is contrary to our intuition about forces. The more massive something is, the more force gravity applies to it to achieve that impartial acceleration. It is almost as if some agency were at work, doing the math and applying proportional force to each measure of mass. Furthermore, though Newton's law of gravitation tells us how to calculate gravitational force (or acceleration), it fails to tell us the reason why it works. The space between the sun and the planets is mostly empty, but still the sun exerts some kind of hold over the planets to keep them from drifting away. For years after this law was formulated, scientists would puzzle over gravity's somewhat mysterious “action at a distance.”

Equally puzzling were the intensive examinations of static electricity and magnetism which followed Newton's explanation of gravity. Certain substances, when rubbed together, will acquire an electric charge. These charges have numerical values and come in opposite types, which we have come to call *positive* and *negative*. The first recorded example of substances which acquire opposing charges when rubbed together is fur and amber. Other examples are wool and plastic; the soles of your shoes on the carpet, or a balloon on your shirt. Having acquired opposite charges, these will attract each other. The balloon, having acquired a charge, will also tend to stick to the wall because it *induces* an opposite charge in some surfaces it touches.

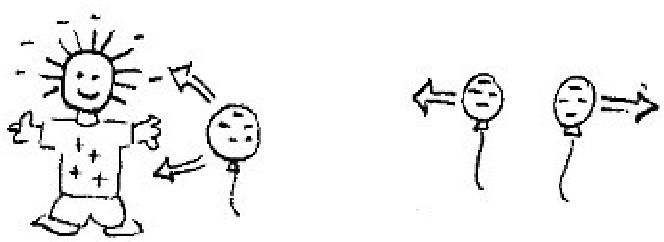

**Figure 6-2.**

Two such balloons, however, having like charges, will repel one another. The reason that children's hair will begin to stand on end after taking repeated trips down the plastic chute on the playground is that their body has acquired a static charge. The individual hairs, having like charge, repel from one another, and this repulsion is strong enough to overcome the downward force of gravity which otherwise would compress them together. Unlike gravity, which always results in attraction between any two masses, static charges and magnets may attract or repel depending on the type – the *polarity* – of the electric charges or the relative orientation of the *poles* of the magnets. Like gravity, the forces arising from static electricity and magnetism are dependent on the square of the distances involved.

In fact, the formula for electrostatic force looks remarkably similar to Newton's formula for gravitation:

**Equation 6-3.** $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$

$\frac{1}{4\pi\varepsilon_0}$ is another constant comparable to G in the formula for gravity. We won't have much to say about it. In place of the masses $m_1$ and $m_2$ in the gravity formula, we have the charges $q_1$ and $q_2$ . And $r^2$ is found in its familiar place as the divisor. But since charges may have positive or negative polarity, the force between them may be positive or negative as well; that is, it may be a force of attraction or repulsion. By contrast, mass is not distinguished by any polarity (positive or negative), so gravity always attracts.

Likewise, the formula for the force between two magnetic poles takes the same form: The force is a constant value times the strength and polarity of each of the poles, divided by the square of the distance between them.

**Equation 6-4.** $F = \frac{\mu}{4\pi}\frac{q_{m1} q_{m2}}{r^2}$

So now we have defined three different kinds of “action at a distance” which vary in force inversely with distance: gravity, magnetism, and static electricity. We may wonder why the square of the radial distance appears in all three cases. Light intensity also happens to vary at the same rate, the further one gets from its source. Our first clue lies in the geometrical formula for calculating the surface area of a sphere:

**Equation 6-5.** $A=4\pi r^2$

The surface area of a sphere is the square of its radius time pi times 4. This factor, $4\pi r^2$ , appears in the formulae for both electrical and magnetic force, above. If we adjust the value of G, it can also appear in the formula for gravity. So let's imagine a sphere with the same radius as the distance between two masses interacting electrically, magnetically, or gravitationally. What does the surface area of this sphere have to do with the strength of the force between them? The larger it is, the weaker the force is. It is as if a particular kind of energy is being radiated by both of them, similar to light, and that this energy is spread out over a greater surface area at greater distances. These masses intercept a proportion of this energy which is proportional to their share of the surface area.

Michael Faraday was no great mathematician. His math skills did not include calculus or trigonometry. But he ranks among the greatest of physicists. Electricity and magnetism were major areas of focus for him, and he liked to consider these invisible forces visually. One of his final contributions to the world of science was the idea that “lines of force” radiated from magnets and electric charges, and that the strength and direction of the forces at any point in space depended on the density and direction of these lines. These lines of force extend over infinite distances, but grow less dense and therefore weaker in an outward direction from their source or origin. Though it was met with skepticism at the time, the idea was brilliant. Instead of supposing that two bodies interacted remotely through empty space, after Faraday's time the scientific world began to see space - “empty” or otherwise – as a *field* composed of numerical values for physical properties which varied continuously from place to place. In this paradigm, each body accelerates according to the properties of its local space and in turn, its properties influence the space around it. The lines of force surrounding a positive charge could be diagrammed this way:

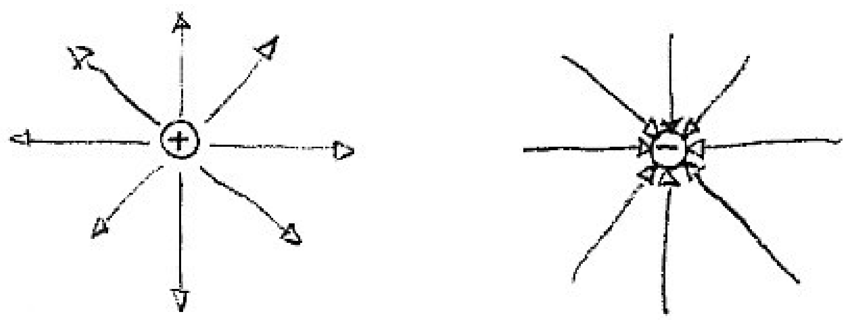

**Figure 6-3.** Lines of force surrounding a positive and a negative charge.

The lines of force (actually *rays*, not lines, since they each have a single direction and originate at a point) for a positive charge point outward and show the direction of the force which would act on another positive charge if it were placed on the line. The lines of force for a negative charge simply point in the opposite direction; a positive charge would be attracted to the negative charge.

The strength of the force is proportional to the density of the lines; more force is felt near the charge than further away because the lines diverge from the charge. Lines of force are not “real” in the sense that they are discrete and can be avoided like cracks in the sidewalk, but they do represent a real intensity and direction of force. The number of lines drawn is arbitrary, although it must be roughly proportional from charge to charge and from place to place.

What we see in Figure 6-3 are electric *monopoles* which are the *origins* of several lines of force. The visual and terminological resemblances to our polar coordinate diagrams in chapter four are not without reason. By placing two like charges, or opposite monopoles, in the same diagram, we see something resembling lines of longitude on a distorted map of the earth:

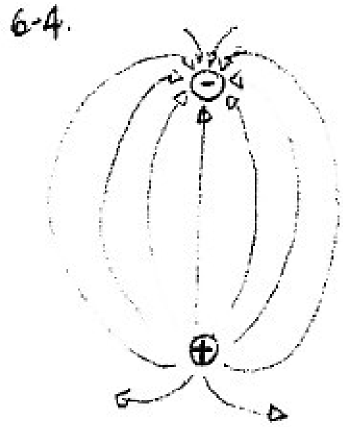


**Figure 6-4.**

One could describe this diagram by saying that the lines of force from each charge bend toward the other charge. The reality that the diagram describes is that there are two competing forces at work and that the balance between the two varies from place to place. A positive “test” charge, if placed in this field, will be forced away from the positive charge and also toward the negative one. Each of these charges exerts a force on the test charge in a direction relative to itself, and these two directions do not necessarily line up. The balance between these forces changes depending on whether the test charge is closer to the positive or negative charge in the diagram.

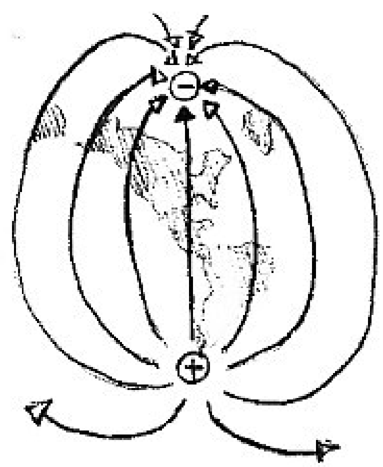

**Figure 6-5.**

Theoretically, all lines of force begin and end on a charge, whether that charge is shown in the diagram or is some distance “off the page,” as with our vanishing points in chapter one. Lines of force always point from a positive charge to a negative one . . . or is that from the south pole to the north? Is Figure 6-5 a map showing the direction of force on charges in electric field or the direction a compass points in earth's magnetic field? As it turns out, electricity and magnetism are related, so sometimes you can use similar pictures for them and just change the labels, like with our fences and railroad tracks in chapter one. We'll explore the relationship of electricity and magnetism more fully in chapter nine .

If we change the relative strength of our opposite charges, the influence of one is greater than the other (Figure 6-6). It also looks like the earth in our distorted map has *rotated* with the stronger pole starting to point toward us (Figure 6-7). The lines of force diverge from electrical charges like parallel lines from a vanishing point on the horizon. The reason that I introduced perspective drawing as the first topic in this book may be growing more apparent to you now. We'll return to this topic several more times.

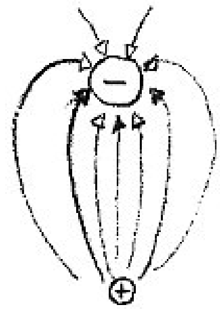

**Figure 6-6.**

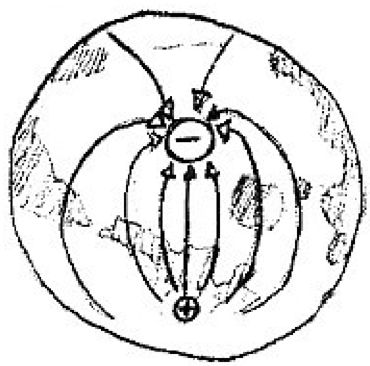

**Figure 6-7.**

In the case of like charges (Figure 6-8), one could say that the lines of force from each charge appear to bend away from the other charge. As in the previous diagram, there are two competing forces at work. A positive “test” charge, if placed in this field, will be forced away from the charge on the left and also away from the one on the right. Each of these charges exerts a force directly away from itself, and these two directions do not necessarily line up. The balance between these forces changes depending on whether the test charge is closer to the left or right charge in the diagram.

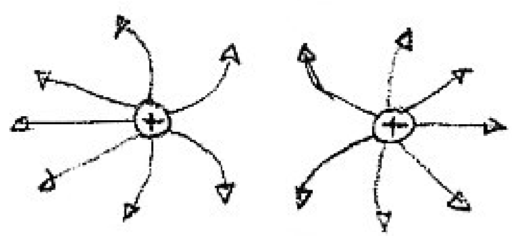

**Figure 6-8.**

One could also suppose that since the lines of force between two opposite charges point in the same direction, they add together and become more dense. Conversely, the lines of force between two like charges point in opposite directions to cancel one another out.

Lines of electric force always originate on a positive charge and *terminate* on a negative one. A difference in electric charge creates something called voltage. Voltage is what creates current, or the flow of electrons, in electric circuits. A battery, for instance, has opposite charges on each of its *terminals*. The greater the difference in charge between the two terminals, the greater the voltage. When a wire is connected to both terminals of the battery, current flows in the wire. The lines of force acting on a positive charge point from positive to negative, but since current is attributed to the flow of negative charges called electrons, we say that current flows from the negative to positive terminals. Thus, in a lines of force diagram, current would tend to flow from the negative charge to the positive one, in the reverse direction of the lines of force. The lines of force show how current would flow in a space of uniform conductivity, such as the open air. Air has very low conductivity; we say that it is a good insulator. If we were to introduce a more highly conductive material into the picture, current would tend to take that path rather than through the air. Placing a metal wire between two opposite charges would be very much like digging a trench between two ponds having different water levels.

Lines of force for magnetic fields are somewhat more complex than those of static charges. “Lines” of magnetic force make closed curves. For magnets, these lines point away from and then always back to their source. They show in which direction a compass needle at a given location would tend to point. Lines of force surrounding a magnetized iron bar having a north and south pole at either end would look like this:

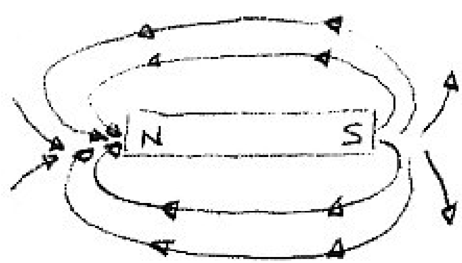


**Figure 6-9.**

A bar magnet is a called a magnetic *dipole;* like the earth, it has a north and south pole and the lines of force run longitudinally between the poles. Unlike a static charge, there can be no such thing as a magnetic monopole. Each line of force always circles back on itself. No matter how you cut the magnet, you will never create independent and opposite poles. If the magnet is cut in half, the two pieces will each have north and south poles. This seems somewhat mysterious at first, and people such as myself may have gone slightly mad trying to figure it out. It is important to realize that the idea of magnetic polarity is based on some false assumptions. There can be no magnetic monopoles because there can't really be dipoles either. Despite what we tend to think, and what I seemed to be suggesting in chapter three, compass needles will never point to a single place in a magnetic field. They will point in a single direction at any given place, but if you follow that direction, you may end up where you began. Let's follow an imaginary compass needle to the earth's north magnetic pole. The compass points us consistently northward, and as we approach the so-called “pole,” the force

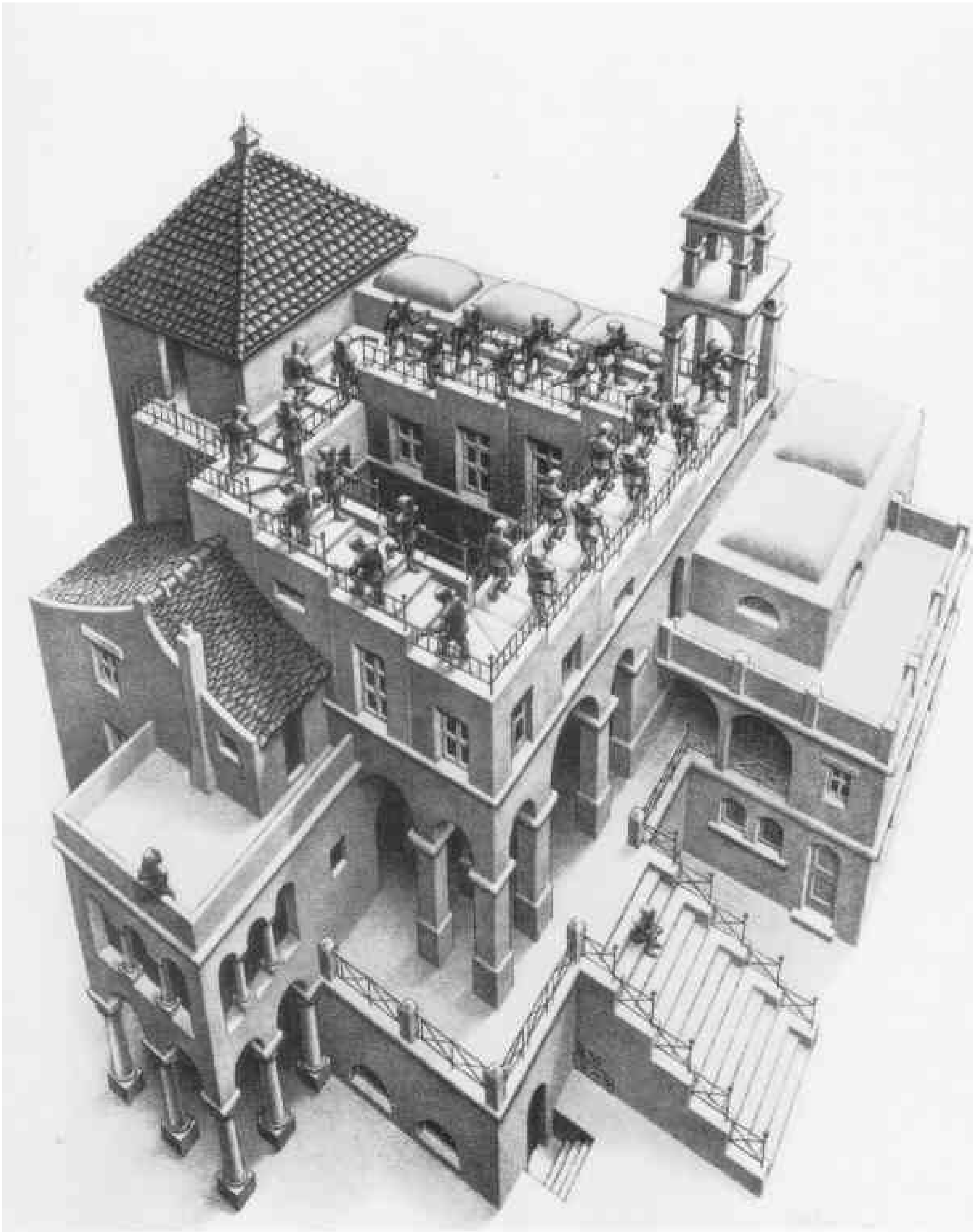

**Figure 6-10.** A staircase with climbing monks by M. C. Escher. One set follows the compass needle south, the other north.

on the needle gets weaker and weaker until we finally arrive and find that the needle drifts aimlessly in every direction. This is not because we have arrived at the “end” of the magnetic lines of force in the way we might follow electric lines of force to their end at a charge. We have simply failed to realize that the earth's magnetic field is three-dimensional. We have been holding our compass level with the ground at all times until it no longer works where it has led us. When we are at the north magnetic pole, which way is “north”? Straight into the ground. Our compass needle has been wanting to point more and more in the direction of the ground during our entire excursion. Let's suppose we could follow the direction of the compass needle through the ground. Where would it take us? To the south pole. But isn't it supposed to point north? The idea or convention of an absolute north and south only applies on the surface and at the lower latitudes. From where we stand, and anywhere inside the earth, the south pole is north of the north pole. If we tunneled through the earth and emerged at the south pole, our compass needle would point straight out of the ground. If we picked any ground-level direction we wanted as “north,” the compass needle would eventually start to steadily point that way. Doesn't that seem odd? By choosing a direction, any direction, as our north, we decide how the compass will behave.

The field surrounding a magnetic “dipole” has a somewhat complex shape, but not all magnetic fields have this sort of apparent polarity. The simplest element in any magnetic field is a single, circular (“*apolar”*) line of force. These circular lines of force can appear in various ways. As we will see in chapter nine, they encircle wires in which electric current is flowing; if the wire is straight, the circles stack up (Figure 6-11). This would make them *electrically* polar but still magnetically *apolar*. But when the wire is circular or helical (Figure 6-12), or in other cases where the circular lines of force have centers which are points on a common circle perpendicular to all of them, the magnetic lines of force form a magnetic dipole.

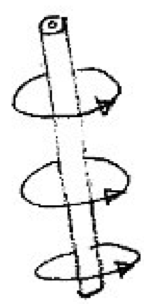

**Figure 6-11.** Circular lines of magnetic force surrounding a conducting wire.

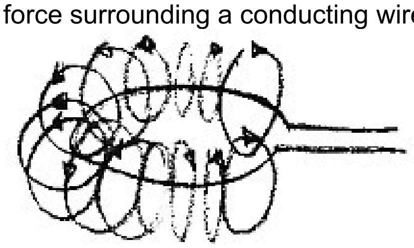

**Figure 6-12.** A toroid (donut-shaped) magnetic field surrounding a loop of conducting wire.

A semicircular, or “horseshoe” magnet is interesting to consider. Instead of forming circles around the magnet, the lines of force go in a straight line from pole to pole. But aren't they supposed to be closed loops? Take another look at the bar magnet (Figure 6-13). Here, the lines of force are almost circular, but the magnet forms the missing section of each circle. In the horseshoe magnet (Figure 6-15), the missing section of the circle is the missing part of the horseshoe magnet. In both cases, the lines of force are circular but part of that circle is through the magnet. In the horseshoe magnet, though, all the circles seem to center around the same place. In the bar magnet, the circles curve out of the ends of the magnet in several directions as if searching for the rest of a circular magnet, a “phantom limb.” A bar magnet is like a circular magnet turned “inside out.” Let's take a bar magnet and twist it “outside-in:”

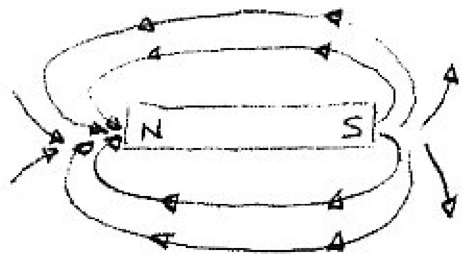


**Figure 6-13.** Bar magnet.

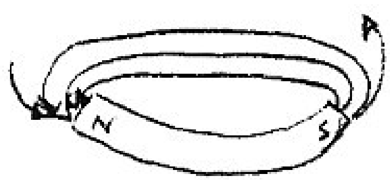


**Figure 6-14.** Bent bar magnet.

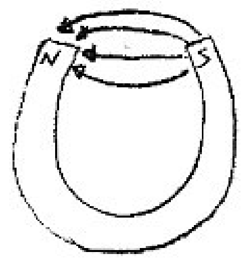


**Figure 6-15.** Horseshoe magnet.

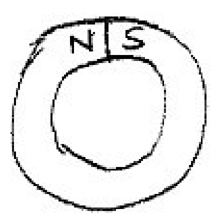


**Figure 6-16.** Horseshoe magnet bent to form a full circle.

Lines of force can also be used to visualize the strength and direction of gravity around various masses. The principles are more or less the same as for opposite charges; the lines do not return to their source and the masses always attract. The most important concepts to take away from this chapter so far are the concept of lines of force and the *inverse square law*, that force varies inversely with the square of the distance from the source.

Lines of force are useful visualizations, but since they are discrete rather than continuous, they do not lend themselves as well to mathematical analysis as the continuous field of numerical values they represent. As a more robust alternative to lines of force, we have a mathematical concept called the *vector*. A vector is a combination of a numerical value and a direction. It can be used to represent any of the directional quantities we have discussed so far, such as distance, velocity, acceleration, and force. Since mass has no direction, it cannot be represented as a vector. We call directionless quantities *scalar* values. The wonderful thing about vectors is that we can combine them mathematically with each other and with scalar values to get results which still have both numerical value and meaningful direction. Each point on our lines-of-force diagrams – even a point between the lines – has a vector associated with it. The vector's direction is the same as the nearby lines of force and its value can be calculated from whichever of the above formulas is applicable. In the case of gravity, we could calculate a vector field around a given mass to give us the acceleration that would be caused on a test mass. That acceleration would have different scales and directions at different points. For another mass in another position, the vector field would differ in scale and direction at each point. If we have calculated each of these two vector fields separately but now want to include the acceleration at some point caused by *both* masses, we need only add the vectors from each independent field to know their combined result. If mass number one's influence at a given point in space causes acceleration of $3m/s^2$ to the right, and mass number two causes acceleration of $7m/s^2$ to the left, the net result is a vector of $4m/s^2$ to the left (Figure 6-17).

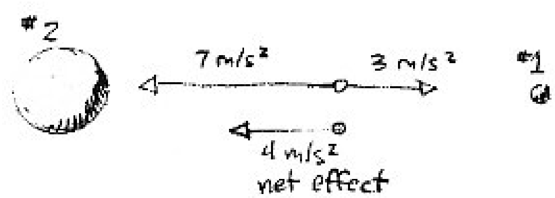


**Figure 6-17.** Accelerations of 7 m/s² in one direction and 3 m/s² in the opposite direction combine for a net acceleration of 4 m/s².

In the above example, the accelerations were in opposite directions, but vector addition is valid for any angle between the vectors. If mass number one causes acceleration of $3m/s^2$ to the right, and mass number two causes acceleration of $4m/s^2$ upward, the net result is a vector having a scale of $5m/s^2$ and a very specific direction which is slightly more upward than to the right.

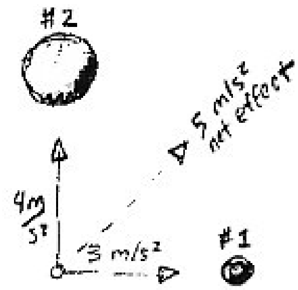


**Figure 6-18.** Accelerations of 4 $m/s^2$ in one direction and 3 $m/s^2$ in a perpendicular direction combine for a net acceleration of 5 $m/s^2$.

Vectors are drawn as arrows, and to add two vectors visually we simply connect the head of one to the tail of the other. The result is a vector which connects the tail of the first to the head of the second. Angles which are not multiples of 90 degrees require trigonometry to solve, but the preceding example illustrates the 3-4-5 case of the Pythagorean theorem. Its visual representation is below:

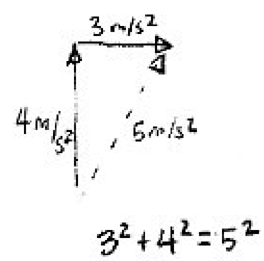


**Figure 6-19.** Vector addition.

Vector mathematics are important for calculating mechanical phenomena that occur in more than one dimension, that is, not along a straight line. A point in either of our two coordinate systems, Cartesian and polar, is a vector from the origin. It has a scale and a direction. In the Cartesian coordinate system, each coordinate of a point is also a vector. The first vector, x, has the length x and is in the direction of the x axis. The second vector, y, is likewise of the length y and in the direction of the y axis, and so on for as many dimensions as the system has. We add these vectors together to calculate the vector which is the point itself.

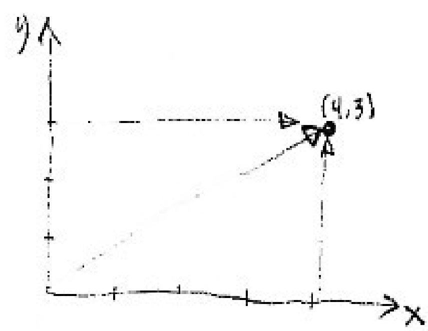


**Figure 6-20.** The vector (4,3) and its components.

In the polar coordinate system, one of the two coordinates is a vector, and the other is a scalar. The first coordinate, *r*, is the radial distance from the origin. It is a scalar value having no direction. To calculate the vector which the coordinate pair represents, we multiply this scalar value with the vector which is the second coordinate, the angle from the axis. The second coordinate, *θ* ("theta"), is a vector whose direction is defined by the angle *θ* and whose length is one. We call this type of vector a *unit vector* because when we multiply it by a scalar, the length of the vector becomes the same value as the scalar.

In Figure 4-5, the point (4,45) is at a radial distance of 4 units and an angular distance of 45 degrees. A vector pointing in the direction of (4,45) but having only one unit of length would be the unit vector specified by the second coordinate. This would be multiplied by the scalar value 4 (the first coordinate) to result in the vector (4,45).

As we go deeper into the role of time in measurement, it will be important to remember that radial distance is a scalar rather than a vector quantity. It is also significant that radial distance is always a positive value, never negative.

Our last major topics in this chapter are *energy* and *momentum* and their relationship to mass. We will learn the meaning of the classical laws that state all three of these quantities must be *conserved*. When we say that mass is conserved, we mean that it can be cut, ground up, melted, boiled, frozen, or dissolved in water, but it will neither be created or destroyed. Momentum is a measurement of a mass times its velocity (*mv*). When we say that momentum is conserved, what we mean is that if you add all the momentum vectors for a group of objects both before and after they exert forces on one another, the sum of their momentum will be the same both before and after. In Figures 6-21 and 6-22 we see the collisions of two one-gram objects. Let's measure their velocities in meters per second. In both cases, the sum of their momentum is two gram-meters per second both before and after the collision.


**Figure 6-21**. Two objects collide and separate. Total momentum remains constant.


**Figure 6-22**. Two objects collide and stick together. Total momentum remains constant.

The conservation of momentum is a fundamental law of physics which holds true for any frame of reference. Let's look at the examples in Figures 6-21 and 6-22 from a different frame of reference, one which is moving 2 m/s to the left relative to the one above. Being in motion, we now measure a momentum of 6 gram-meters per second in this system, but we note that this value does not change during the collision:


**Figure 6-23.** Two objects collide and separate. Total momentum remains constant.

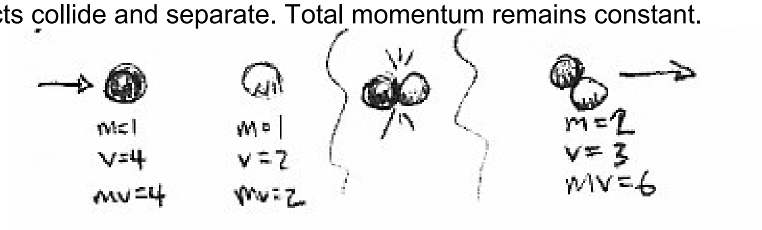


**Figure 6-24.** Two objects collide and stick together. Total momentum remains constant.

The conservation of momentum is simply Newton's Second Law (*F=ma*) in disguise. When two masses interact, they exert equal and opposite forces on each other which result in acceleration, which is a change in velocity. Those velocity changes will be in opposite directions, so the change in average velocity of the system as a whole is *zero*. The aggregate momentum is the same before and after. We can rewrite Newton's second law by multiplying both sides by time, or the duration of the applied force:

$$F = ma \qquad a = \frac{v}{t}$$

$$F = m\frac{v}{t}$$

$$Ft = mv$$

Changes in momentum are like the dollars we passed to Jane in chapter three; our loss was her gain. The total amount of dollars was conserved. Momentum is conserved in the same sense. Momentum in two dimensions is more difficult to calculate, but is still conserved, as in the example below.

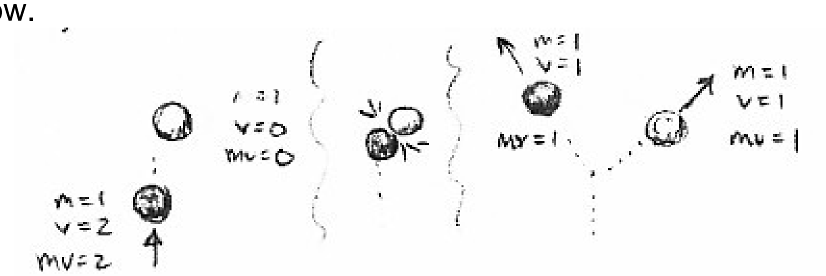


**Figure 6-25.** A collision in two dimensions. The objects strike in a glancing blow rather than head-on.

The examples shown so far are of linear momentum, that is, mass having velocity in a straight line. We have discussed three kinds of distance so far: linear, angular, and radial. Likewise there are three corresponding types of velocity: linear, angular, and radial. Angular velocity is the rate of an object's rotation or revolution. Linear velocity is measured as linear distance per unit of time (meters or feet per second) and is a vector quantity; radial velocity is measured as radial distance per unit of time and is a scalar quantity because radial distance is also scalar; angular velocity is measured as angular distance per unit of time (degrees per second) and is a vector. Playback speed for 20th-century phonograph records was specified in terms of angular velocity. The three most common speeds were 78, 45, and 33 1/3 revolutions per minute ("RPM"). The earth has an angular velocity of roughly 15 degrees per hour, which means that it makes a full 360 degree rotation about once every 24 hours. Even if we disregard the earth's motion around the sun, it still has a momentum due to this rotation. It is *angular momentum*, and the earth's angular velocity can only be changed as a result of applied force.

Any rotating mass has angular momentum. In order to be able to express the direction of this momentum simply, we use the object's one-dimensional axis of spin rather than the two-dimensional circular movement of any given part of the object. For angular phenomena, we arbitrarily pick a system of "handedness" to distinguish clockwise spin from counterclockwise spin. In a right-handed system, counterclockwise spin (look at the direction in which the fingers of the right hand curl) is represented as a vector pointing in the direction from which the spin appears counter-clockwise (if the fingers of your right hand appear to curl in a counter-clockwise direction, your extended thumb will point at you). A *left*-handed system (on the other hand) would represent *clockwise* spin by the same direction. Either system is valid, but one system or the other is usually agreed upon as a convention for the sake of communication.

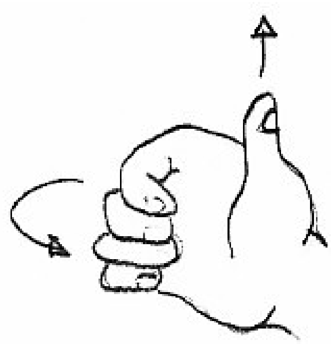

**Figure 6-26**. In a right-handed system, spin in the direction of the fingers of the right hand (counterclockwise) is represented by a vector in the direction of the right thumb (upward).

Even though we are in the middle of talking about conservation, this seems a reasonable place to explain something important about angular velocity. The earth spins around once every day. Another word for spin is rotation. The earth rotates three hundred and sixty degrees every day; in mathematical terms, then, its angular velocity is about three hundred and sixty degrees per day. More precisely, its angular velocity is 359 degrees per 24 hours. The earth's rotational speed, or angular velocity, is the main reason we see the rising and setting of the sun. In addition to the earth's *rotation* is its *revolution* around the sun once per year; this is an angular velocity of 360 degrees per year, or about 1 degree per 24 hours. The earth's revolution around the sun causes the sun to rise and set one additional time every year. We discussed a similar problem in Mr. Fogg's story. His angular velocity eastward around the earth increased his perception of the angular velocity of the sun. Since he and the sun were traveling in opposite directions, he might have subtracted the angular distance of his travels (1 full rotation) from the sun's apparent angular distance (80 rotations) to calculate the actual number of elapsed days in his journey (79 days). In the case of the earth's journey around the sun, we add the angular velocities due to the earth's rotation (359 degrees per 24 hours) and revolution (1 degree per 24 hours) to get the 24-hour *average* periodicity of the rising and setting sun (360 degrees per 24 hours). The reasons for daily variations from this average relate to many key points in this book, and are discussed in Appendix A.

Notice that when we add and subtract angular measurements, we do not distinguish rotation from revolution. The earth is a rigid body which is rotating. Its individual parts are revolving around its center. Mr. Fogg made one full revolution around the earth, and the earth revolves around the sun. The rotation, or spin, of a rigid body cannot be distinguished from revolution; The earth's revolution requires the *external* centripetal force of the sun's gravity to sustain its orbit; in the case of the earth's rotation, the centripetal force is *internal*. Both motions are qualitatively equivalent as angular velocities.

Another reason that revolution need not be distinguished from rotation is found in the following thought experiment. Imagine a circular line on the ground. You follow this line in a clockwise direction and for each time you revolve around the circle's center, you undergo one full rotation as well. At the north side of the circle, you are facing east and on the south side you are facing the opposite direction, west. If you are brave enough to do the trigonometry or perceptive enough to do without the calculations, you will notice a sine/cosine relationship between the compass point you stand on and the direction you are facing, but that is not the point being made. The point is that no matter how small or large the circle is, one revolution around the circle causes you to undergo exactly one rotation. As the imaginary circle's size approaches zero, you will find yourself simply rotating in place.

Lastly, we come to the topic of the conservation of energy. Turning off the lights when you leave a room is conserving energy, but in physics we mean something else: that energy is neither created nor destroyed. Energy takes many forms, and may be transformed from one type to another, but (in classical physics at least) the total energy of a system remains constant, and this is what is meant in physics by the conservation of energy. A falling stone has what is called kinetic energy. Kinetic energy is calculated as one half times mass times velocity squared. When the stone hits the ground, that kinetic energy is partly passed into the air and ground as sound waves, and partly transformed into heat. When the stone is suspended above the ground, we say in classical terms that it has *potential energy* so we can explain where the kinetic energy comes from when we let it drop. If this seems to you like cheating or fudging, I can't say that I disagree; but gravitation is a subject beyond the scope of this book. So is the relationship of mass and energy; in modern physics, energy and mass are known to be interchangeable, so that mass and energy are not necessarily individually conserved in a system, but they are conserved as a combined total.