Chapter 8
Motion in Perspective

As we study what happens to our perspective as we move from place to place, we will be continuing several themes. We will be distinguishing the obvious from the apparent; what we measure from what we see; the Cartesian from the polar; and the global from the local.

Where I grew up, Interstate 5 crosses through wide expanses of farmland. Take an imaginary trip with me in the car through the Skagit Valley. We're sitting in the passenger seats, so we're free to look idly about in any direction. Out the right-hand window, vast acres of silt deposited in this valley long ago by the Skagit River are freshly tilled into long furrows, which in our case happen to be running perpendicular to our section of freeway. Looking down these furrows out the side window, the near ends whip past us at 70 miles per hour, while the far ends hardly appear to move at all. An illusion is created that this plowed field is rotating in a counterclockwise fashion, with the center of rotation being some indeterminate but long distance away. Conversely, we might see the illusion of clockwise motion out the opposite window. But is this rotation purely illusion? As we drive north and approach a barn on our right, we see the south side. As it passes us, we see the west side. Looking behind us, we then see the north side. Let's just say for now that the illusion is stubbornly persistent.

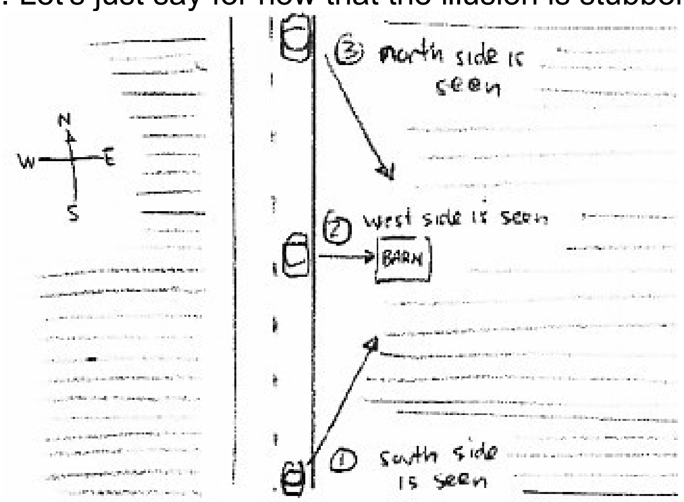

Figure 8-1.

Rotation has a related geometrical concept called *curl*. When an object is rotating, it has an angular velocity which is uniform at every point, inside and out. In other words, each part of the object is circling the axis of spin with the same frequency. The linear velocities of each of the parts, however, vary greatly in both direction and speed. The outside moves faster than the inside; one side moves in the opposite direction of the other. Let's look at a wheel as an example.

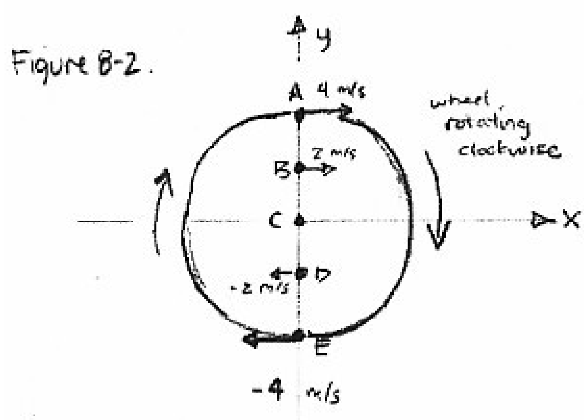

Figure 8-2.

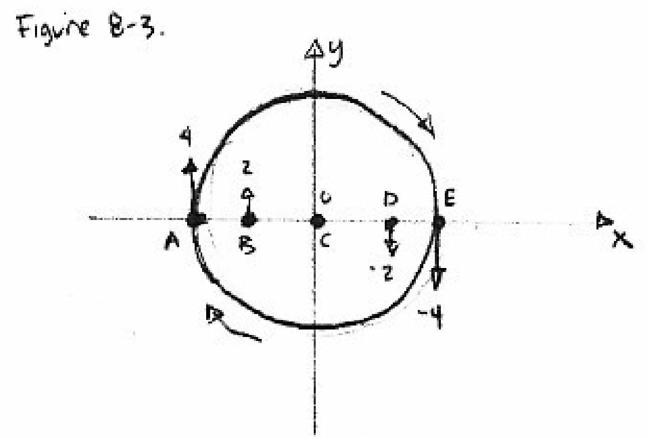

Figure 8-3.

In Figure 8-2, point A on the wheel has a linear velocity vector of four feet per second along the x axis. Point B has a velocity vector of two feet per second in the same direction. Point C has no linear velocity at all, and the points shown on the opposite sides of the wheel have negative velocities. There is a continuous range of linear velocities from point A to point E with values from 4 to -4. There are an equal range of velocities relative to the y axis, as seen in Figure 8-3. Now let's look at a smaller section of Figure 8-2, the area between points A and B.

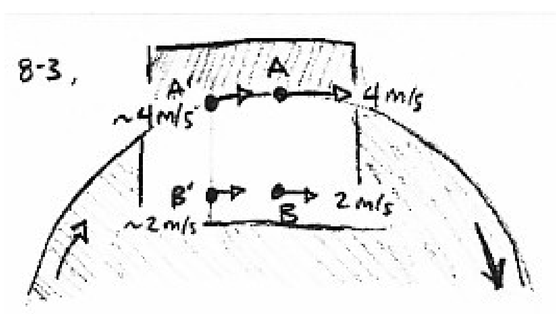

Figure 8-4.

Taking very simple approximations, the difference in linear velocity between the top of this picture and the bottom is two feet per second. The difference between the left and right sides is much more subtle. When we pick a very small space and see more difference in a vector from one pair of opposite sides than difference in another pair of opposite sides, the vector field is nonuniform. When the difference in vector strength varies in a direction perpendicular to the vectors, we say that we measure *curl* (Figure 8-5). The precise formula for curl measurement can be found in Appendix B.

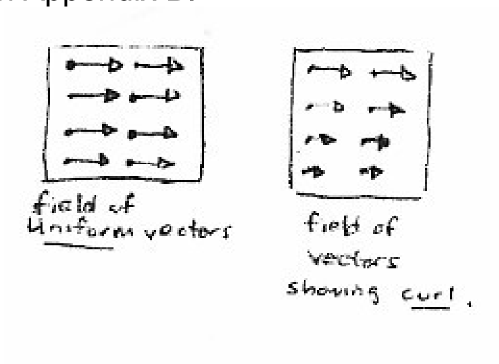

Figure 8-5.

Like our car on the freeway, we may only notice a difference along one axis (rather than the two axes of motion we see in rotational movement) and we may only see movement in one direction (rather than the opposite directions of rotational movement). Measurement of curl tells us that there *might* be overall rotation in the bigger picture, but there might not be. Curl has a numerical value and may be positive or negative, much like clockwise or counterclockwise rotation may be a vector with positive or negative value. We might imagine that we are seeing positive curl out one window and negative curl out the other. Indeed, we would see this effect in any direction perpendicular to our line of motion (through the sunroof, for instance) if we had regular points of reference to look at (very tall telephone poles, perhaps). Later on, we'll figure out whether this apparent curl has a basis in mathematical reality.

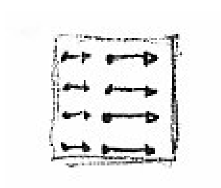

Figure 8-6. Field of vectors showing divergence.

Closely related to curl is the measurement of *divergence*. In the case of divergence, the vector strength varies in the direction of the vectors. The formula for calculating divergence can be found in Appendix B. We see the appearance of divergence out the front and rear windows of the car. If two telephone poles face each other from either side of the road, they appear to diverge, or get further apart, as we approach, and to converge as they get further behind us. In mathematical terms, there appears to be positive divergence ahead of us and negative divergence behind us. This is indeed an illusion with no mathematical basis. No measurable divergence is created by our motion. We might say that any divergence we observe is there whether we are moving or not.

Our motion does not cause the plowed field to rotate, nor does it create curl among the linear velocities of the points in the field. However, there is one way in which the curl is mathematically real. Our motion relative to the field gives all the points in the field a uniform linear velocity, but a *nonuniform* angular velocity. Depending on the location of a point in the field, that point is changing its angle in our polar frame of reference at a different rate.

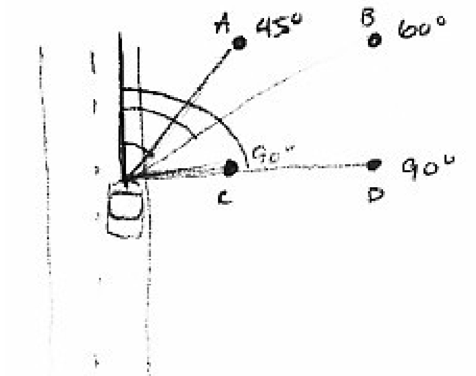

Figure 8-7. When our car is stopped, all points in the field remain at the same angle in our polar coordinate system.

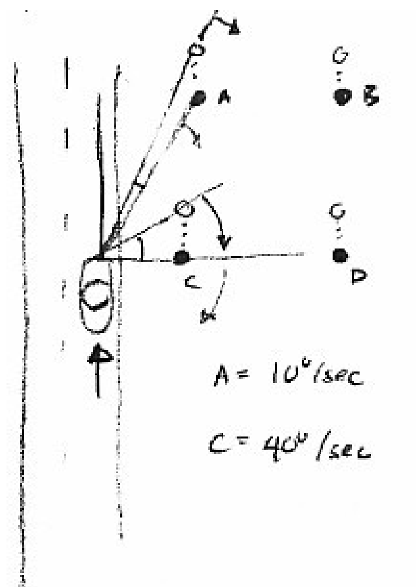

Figure 8-8. When we are in motion, different points in the field change their angle at different rates.

Angular velocities vary in every direction in this plane: both from near to far and from beside us to ahead of us. Remember that angular velocity is measured as a vector perpendicular to the plane of rotation. So our field is a field of up-or-down vectors which vary continuously in all horizontal directions (Figure 8-9). It happens that these vectors of angular velocity do not vary along the direction in which they point, which is vertically. This situation satisfies the mathematical definition of curl.

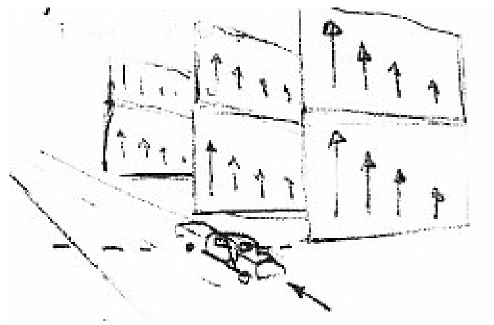

Figure 8-9. A three-dimensional field of angular velocity vectors, showing curl.

The two points to remember in all of this is that relative motion does not create divergence, but it does create a vector field of angular velocities, and this field has curl. When our relative velocity is zero, this curl is zero. We'll see why these two points are significant when we discuss the equations relating electricity and magnetism in the next chapter.

Actually, there is one more point to be made from our imaginary freeway trip. As we look out the side window, the angular velocity of each point in the field in our line of sight varies inversely with the square of its distance from us. Now where have we seen that relationship before?

Before we move on to electromagnetism, we're going to revisit a few things with our new friends divergence and curl. First we'll return to electric and magnetic force. When we draw lines-of force diagrams for magnetism and static electricity, these two forces look quite different. Electric lines of force *diverge* from a static charge; they grow stronger or weaker, more or less dense, in the direction which they point. Magnetic lines of force *curl* through a magnet; they grow stronger or weaker, more or less dense, perpendicularly to the direction which they point.

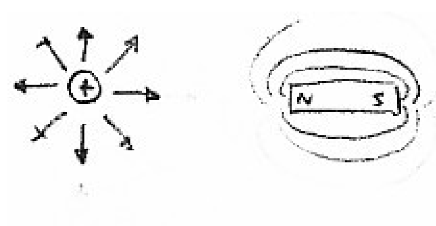

Figure 8.10.

Let's also return for a moment to our two rules for perspective drawings:

Rule #1: A series of regularly spaced objects will appear to come closer together as the series recedes toward the horizon.

We will correlate this rule to the concept of curl.

Rule #2: Horizontal lines, if they point in the same compass direction, diverge from a common point on the horizon called their “vanishing point.”

This correlates to the concept of divergence.

To illustrate, let's look at a drawing of three sets of railroad tracks, spacing each set of tracks from each other at the same distance the rails have within each set. First we'll look at just the rails as they come out of the vanishing points. If this was a lines-of-force diagrams rather than perspective drawing, this would show divergence. The crossties, getting closer together as they approach the horizon, would represent curl if they were lines of force.

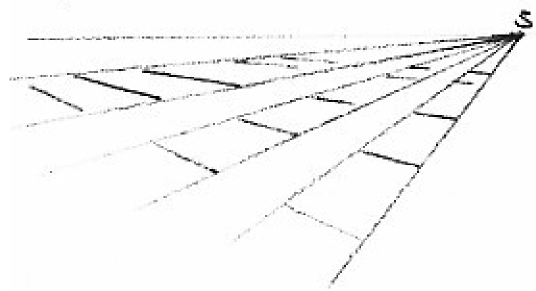

Figure 8-11.

If we make this a wide-angle drawing, the rails and cross ties both show curl or divergence depending on which section of the drawing you look at. The rails *diverge* near the vanishing points but they *curl* midway between them; that is, they curve and are parallel but are spaced closer together nearer the middle of the horizon. The cross ties show “curl” at the vanishing points (even though they are not curved, they are closer together), and at the middle of the drawing, they no longer look parallel; they diverge.

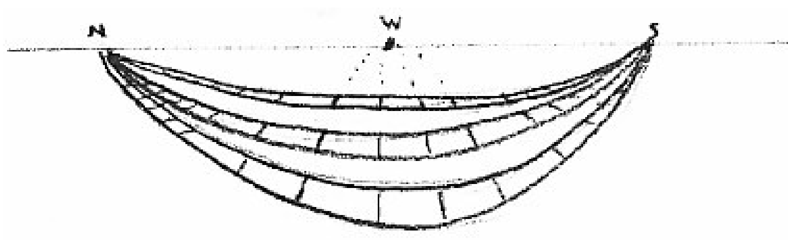

Figure 8-12.

What happens if we animate this scene? Let's run alongside the railroad tracks. The tracks are pretty much the same at every point, so their position in the drawing does not change as we run parallel to them. But the crossties are moving, aren't they? Watch them move down the tracks like they're items on a conveyor belt. Can you see them in the drawing, appearing to rotate as they round the artificial curve in the straight tracks? They look just like the furrows in the field as we passed them on the freeway.

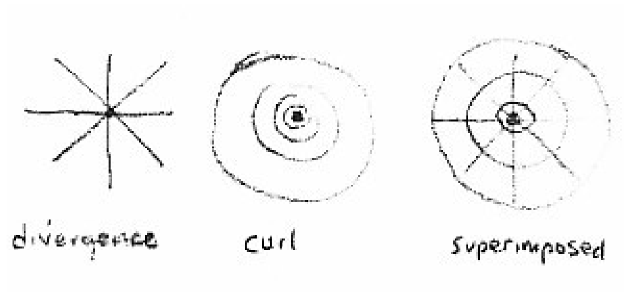

Figure 8-13.

Lastly, do you remember our light cones from chapter two? Divergence and curl, diagrammed above, also combine to give us a cone shape (Figure 8-13). Look at the third image above and visualize looking down the center of a cone with one set of regularly-space lines drawn from tip to edge and another set of curves which cross each of the first set of lines.

Take some time to ponder this chapter and observe for yourself. Take a ride and look out the window!